Optimal Soot Blowing Strategies in Boiler Systems with Variable Steam Flow

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Outline

The optimization of boiler soot blowing is valuable not only from an economic point of view, but also from the perspective of environment.

Contradiction

- If the operations of soot blowing are performed more frequently, thinner soot leads to higher efficiency.
- The frequent operation of soot blowing will also give rise to a waste of steam and increased maintenance cost.

The goal of optimizing soot blowing for boiler systems is to optimize the frequency of soot blowing or **the start time and end time of soot blowing** so as to minimize the combined cost of fouling and soot blowing operations.

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The most related literatures works focused on the establishment of boiler model, fouling prediction and fouling assessment:

- Mathematical model
- Expert system
- Support Vector Machine
- Artificial Neural Networks
- Adaptive Neuro-Fuzzy Inference Systems

We focus on the optimization of soot blowing via HJB method.

- Central of optimal control theory
- Necessary and sufficient condition.
- Generalize to **stochastic systems**.

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Mathematical model of boiler soot blowing

- The boiler operation modes are classified as soot deposition, denoted by 1, and soot blowing, denoted by 2.
- The boiler **cycles** between mode 1 and mode 2.
- Continuous time Markov process is constructed as the dynamics of boiler.

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Mathematical model of boiler soot blowing

The transition rate $\lambda_{\alpha\beta}$ from mode α to mode β satisfies the conditions

$$\begin{cases} \lambda_{\alpha\beta} \geq 0, \text{ if } \alpha \neq \beta \\ \lambda_{\alpha\alpha} = -\sum_{\alpha\neq\beta} \lambda_{\alpha\beta}, \text{if } \alpha = \beta \end{cases} \quad (1)$$

Correspondingly, the transition rate matrix Q is defined as

$$Q = \begin{bmatrix} -\lambda_{12}(t) & \lambda_{12}(t) \\ \lambda_{21}(t) & -\lambda_{21}(t) \end{bmatrix} = \begin{bmatrix} -\omega_d(t) & \omega_d(t) \\ \omega_b(t) & -\omega_b(t) \end{bmatrix} \quad (2)$$

where, $\lambda_{12}(t) = \omega_d(t)$, $\omega_d^{-1}(t)$ is the soot deposition time; $\lambda_{21}(t) = \omega_b(t)$, $\omega_b^{-1}(t)$ is the soot blowing time.

Mathematical model of boiler soot blowing

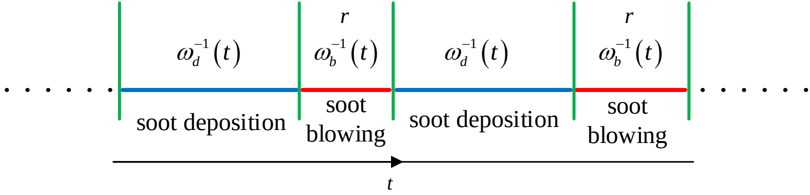
The transition probability matrix P is given by

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} \quad (3)$$

where,

$$p_{\alpha\beta} = \mathrm{P}\left(\Theta\left(t+\delta t\right) = \beta | \Theta\left(t\right) = \alpha\right) = \begin{cases} \lambda_{\alpha\beta}\delta t + o\left(\delta t\right), & \alpha \neq \beta \\ 1 + \lambda_{\alpha\beta}\delta t + o\left(\delta t\right), & \alpha = \beta \end{cases}$$

and $\lim\limits_{\delta t \to 0} \frac{o(\delta t)}{\delta t} = 0$ for all $\alpha, \beta \in B$.



Mathematical model of boiler soot blowing

The **soot thickness** of boiler is denoted as $x(t)$, and satisfies the following state equation

$$\dot{x}(t) = d(t) - b(t) \tag{4}$$

where, $b(t)$ is soot blowing rate and written as

$$b(t) = \begin{cases} 0, & \Theta(t) = 1 \\ k_r r(t), & \Theta(t) = 2 \end{cases} \tag{5}$$

$d(t)$ is soot deposition rate and written as

$$d(t) = \begin{cases} \xi e^{-\mu x(t)}, & \Theta(t) = 1 \\ 0, & \Theta(t) = 2 \end{cases} \tag{6}$$

In our model, **soot thickness** is selected as the system state; **soot deposition time**, **soot blowing time** and **steam flow** are the three control variables.

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HJB Equation

The cost function is constructed as

$$J\left(\alpha,x,\omega_d,\omega_b,r\left(t\right)\right)=\mathrm{E}\left\{\int_0^\infty e^{-\rho t}G\left(\alpha,x,\omega_d,\omega_b,r\left(t\right)\right)dt \middle| x\left(0\right)=x,\Theta\left(0\right)=\alpha\right\} \tag{7}$$

where, ρ is discount rate and

$$G\left(\alpha,x,\omega_d,\omega_b,r\left(t\right)\right)=c_b r\left(t\right)\mathrm{Ind}\left(\Theta\left(t\right)=2\right)+g\left(x\left(t\right)\right)$$

$$c_b=Kk_r$$

$$\mathrm{Ind}\left(\Theta\left(t\right)=\alpha\right)=\begin{cases}1,\text{if } \Theta\left(t\right)=\alpha\\0,\text{otherwise}\end{cases},\alpha\in B$$

$$g\left(x\left(t\right)\right)=c_d x\left(t\right)$$

HJB Equation

Based on $J(\alpha, x, \omega_d, \omega_b, r)$ and the mathematical model, the problem of soot blowing optimization is described as :

For the given soot thickness x and boiler mode α in the initial time, to obtain the control policy (ω_d, ω_b, r) in the set of admissible control policies

$$\Gamma(\alpha) = \{(\omega_d(\cdot), \omega_b(\cdot), r(\cdot)) \in \mathbb{R}^3 : \omega_d^{min} \le \omega_d(\cdot) \le \omega_d^{max}, \omega_b^{min} \le \omega_b(\cdot) \le \omega_b^{max}, r_{min} \le r(\cdot) \le r_{max}\} \quad (8)$$

so as to minimize the cost function $J(\alpha, x, \omega_d, \omega_b)$ subject to the constraints given by (1)-(6). Namely, the goal of this paper is to solve the following optimal problem

$$\begin{cases} \min\limits_{(\omega_d, \omega_b, r) \in \Gamma(\alpha)} & J(\alpha, x, \omega_d, \omega_b, r) \\ s.t. & (1)-(6) \end{cases}$$

HJB Equation

$J(\alpha, x, \omega_d, \omega_b, r)$ can be rewritten as*

$$J(\alpha, x, \omega_d, \omega_b, r) = \int_0^\infty e^{-\rho t}\Big(G(\alpha, x, \omega_d, \omega_b, r) + \sum_{\beta \in B} \lambda_{\alpha\beta} J(\beta, x, \omega_d, \omega_b, r)\Big) dt \tag{9}$$

and the corresponding value function is as follows

$$v(\alpha, x) = \inf_{(\omega_d, \omega_b, r) \in \Gamma(\alpha)} J(\alpha, x, \omega_d, \omega_b, r), \forall \alpha \in B, x \in \mathbb{R} \tag{10}$$

Regarding the optimality principle, HJB equations can be written as

$$\rho v(\alpha, x) = \min_{(\omega_d, \omega_b, r) \in \Gamma(\alpha)} \Big\{ G(\alpha, x, \omega_d, \omega_b, r) + \sum_{\beta \in B} \lambda_{\alpha\beta} v(\beta, x) + v_x(\alpha, x)(d(t) - b(t)) \Big\} \tag{11}$$

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Let control policy $u = (\omega_d, \omega_b, r)$, then $J(\alpha, x, \omega_d, \omega_b, r) = J(\alpha, x, u)$, $G(\alpha, x, \omega_d, \omega_b, r) = G(\alpha, x, u)$. The elementary properties of value function v include:

- v is **convex** for x.
  - $J(\alpha, x, u)$ is convex.
  - v is convex for x.
- $v(\alpha, x)$ is **locally Lipschitz** for x.
  - $G(\alpha, x, u)$ is locally Lipschitz.
  - $|v(\alpha, x_1) - v(\alpha, x_2)| \leq \tilde{C}\left(1 + |x_1|^k + |x_2|^k\right)|x_1 - x_2|$.

Theorem 1

The HJB equation (11) *has a single viscous solution, and v is the single viscosity solution of the HJB equation.*

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Proof.

The proof contains two parts:

1. v is a viscosity solution of HJB equation.
   - $v(\alpha, x)$ is continuous and $|v(\alpha, x)| \leq C\left(1 + |x|^k\right)$.
   - v should be both a viscosity subsolution and a viscosity supersolution.
2. HJB equation has unique viscosity solution.
   - $G(\alpha, x, u)$ is locally Lipschitz and $|G(\alpha, x, u)| \leq C\left(1 + |x|^k\right)$.
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Let variable h represent the length of the finite difference interval of the soot thickness x, then the first-order derivatives of the value function $v_x(\alpha,x)$ can be approximated as

$$v_x(\alpha,x)=\begin{cases}\frac{1}{h}\left(v^h(\alpha,x+h)-v^h(\alpha,x)\right), & \text{if } d(t)>b(t)\\ \frac{1}{h}\left(v^h(\alpha,x)-v^h(\alpha,x-h)\right), & \text{otherwise}\end{cases} \tag{12}$$

Place (12) into (11), one can obtain

$$v^h(\alpha,x)=\min_{u\in\Gamma^h(\alpha)}\left\{\frac{G(\alpha,x,u)}{\Omega^h(\alpha,u)+\rho}+\frac{1}{1+\frac{\rho}{\Omega^h(\alpha,u)}}\cdot\left(\begin{array}{l}\sum_{\beta\neq\alpha\in B}p^\beta(\alpha,u)v^h(\beta,x)\\ +p_x^+(\alpha,u)v^h(\alpha,x+h)\operatorname{Ind}(d(t)-b(t)>0)\\ +p_x^-(\alpha,u)v^h(\alpha,x-h)\operatorname{Ind}(d(t)-b(t)\leq 0)\end{array}\right)\right\} \tag{13}$$

where, $\Omega^h(\alpha,u)=|\lambda_{\alpha\alpha}|+\frac{|d(t)-b(t)|}{h}$, $p^\beta(\alpha,u)=\frac{\lambda_{\alpha\beta}}{\Omega^h(\alpha,u)}$, $p_x^+(\alpha,u)=\frac{d(t)-b(t)}{h\Omega^h(\alpha,u)}$, $p_x^-(\alpha,u)=\frac{b(t)-d(t)}{h\Omega^h(\alpha,u)}$.

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Theorem 2

If $v^h(\alpha,x)$ is a solution of HJB equation (13), and there exists constant C_g and κ_g such that $0 \le v^h(\alpha,x) \le C_g\left(1+|x|^{\kappa_g}\right)$, then $\lim_{h\to 0} v^h(\alpha,x) = v(\alpha,x)$

For the control policy Υ, define the operators T_Υ and T^* which act on $v^h(\alpha,x)$ as

$$T_\Upsilon\left(v^h(\alpha,x)\right) = \frac{G(\alpha,x,\Upsilon)}{\Omega^h(\alpha,\Upsilon)+\rho} + \frac{1}{1+\frac{\rho}{\Omega^h(\alpha,\Upsilon)}} \cdot \left(\begin{array}{l} \sum\limits_{\beta\neq\alpha\in B} p^\beta(\alpha)\, v^h(\alpha,x) \\ +p_x^+(\alpha,\Upsilon)\, v^h(\alpha,x+h)\,\mathrm{Ind}\left(d(t)-b(t)>0\right) \\ +p_x^-(\alpha,\Upsilon)\, v^h(\alpha,x-h)\,\mathrm{Ind}\left(d(t)-b(t)\le 0\right) \end{array} \right) \quad (14)$$

$$T^*\left(v^h(\alpha,x)\right) = \min_{\Upsilon\in\Gamma^h(\alpha)} \left\{ T_\Upsilon\left(v^h(\alpha,x)\right) \right\} \quad (15)$$

Then, the equation (13) can be solved by Kushner's Method.

Algorithm 1 Kushner's Method

Step 1: Set $\varepsilon \in \mathbb{R}^+$, where $\mathbb{R}^+$ represents the set of positive real numbers. $n := 1$, $\left(v^h(\alpha,x)\right)^n := 0, \forall \alpha, x$.

Step 2: Let $\left(v^h(\alpha,x)\right)^{n-1} := \left(v^h(\alpha,x)\right)^n, \forall \alpha, x$.

Step 3: Determine Υ^n so that

$$\begin{array}{l} T_{\Upsilon^n}\left(v^h(\alpha,x)\right)^{n-1} = T^*\left(v^h(\alpha,x)\right)^{n-1} \\ \left(v^h(\alpha,x)\right)^n = T^*\left(v^h(\alpha,x)\right)^{n-1} \end{array}, \forall \alpha, x$$

Step 4: Calculate

$$\bar{c} := \min_{\forall \alpha, \forall x} \left\{ \left(v^h(\alpha,x)\right)^n - \left(v^h(\alpha,x)\right)^{n-1} \right\}$$

$$\underline{c} := \max_{\forall \alpha, \forall x} \left\{ \left(v^h(\alpha,x)\right)^n - \left(v^h(\alpha,x)\right)^{n-1} \right\}$$

$$c_{min} := \frac{\rho}{1-\rho}\bar{c}, c_{max} := \frac{\rho}{1-\rho}\underline{c}$$

If $\left|c_{max} - c_{min}\right| \leq \varepsilon$, stop, $\Upsilon^* = \Upsilon^n$; otherwise, $\left(v^h(\alpha,x)\right)^n = T_{\Upsilon^n}\left(v^h(\alpha,x)\right)^{n-1}$, $n = n+1$, return to **Step 2**.

The experiment parameters are shown in the following Table.

Symbols	Description	Value
ε	criterion	0.001
h_x	step size of soot thickness	0.02
ρ	discount rate	0.05
x_{min}	minimum thickness of soot	0
x_{max}	maximum thickness of soot	10
r_{min}	minimum steam flow	0.5
r_{max}	maximum steam flow	2.5
h_r	initial step size of steam flow	0.1
k_r	coefficient of soot blowing	0.1
ξ	coefficient of soot deposition	0.5
μ	coefficient of soot deposition	0.1
c_d	cost per unit time caused by soot deposition	0.5
ω_{bmin}	reciprocal of soot blowing maximum time	0.1
ω_{bmax}	reciprocal of soot blowing minimum time	10
ω_{dmin}	reciprocal of soot deposition maximum time	0.01
ω_{dmax}	reciprocal of soot deposition minimum time	1.0
K	cost coefficient of soot blowing	5.0
$h_{\omega d}$	initial time step size of soot deposition	0.01
$h_{\omega b}$	initial time step size of soot blowing	0.01

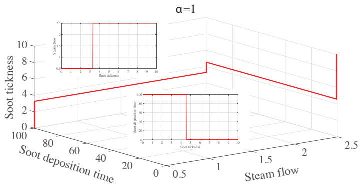

Figure 1: Strategies under the parameters setting

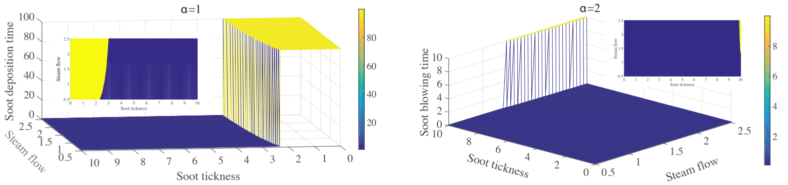

Figure 2: Taking steam flow r in $[0.5, 2.5]$ at each 0.05 interval

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- Propose a cost function and derive the HJB equation.
- Prove the elementary properties of value function.
- Apply Kushner's method to solve the HJB equation.
- Verify the effectiveness of the proposed method via numerical experiments.

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Thanks for your attention!

Q&A